بسمه‌تعالی

Finite Element Method

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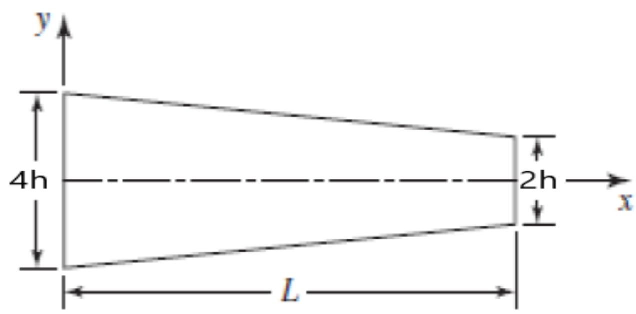
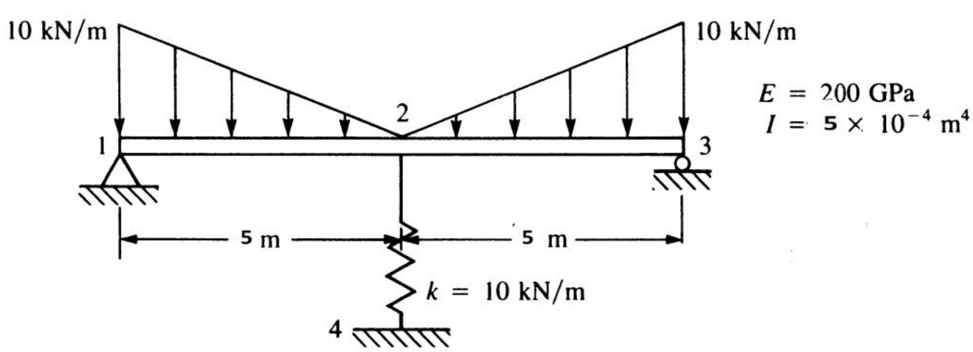
Homework #5	**Beam element**	

1. The tapered beam element shown in the figure has uniform thickness t and varies linearly in height from $4h$ to 2h.
   a) Derive strain energy expression for the element.
   b) Derive the value of component k_{22} of the element stiffness matrix by first theorem of Castigliano.



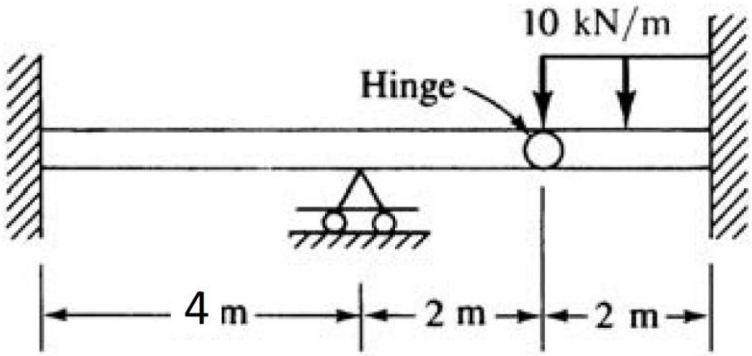
Ref: Fundamentals of Finite Element Analysis, David V. Hutton, 2004

2. For the beams shown in the figure, determine the displacements and the slopes at the nodes, the forces in each element, and the reactions.



Ref: The First Course in the Finite Element, Logan, 4th Edition

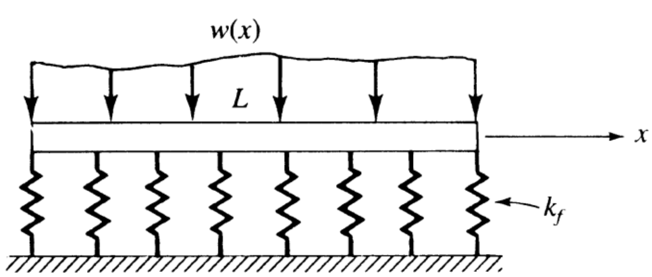
3. For the beams shown in the figure, with internal hinge, determine the reduced stiffness matrix: Let E = 215 GPa and I = 3×10^{-4} m^4.



Ref: The First Course in the Finite Element, Logan, 4th Edition

4. Derive the equations for the beam element on an elastic foundation (Show in Figure) using the principle of minimum potential energy. Here kf is the subgrade spring constant per unit length. The potential energy of the beam is:

$$\pi_p = \int_0^L \frac{1}{2} EI(v'')^2\, dx + \int_o^L \frac{k_f v^2}{2}\, dx - \int_0^L wv\, dx$$



Ref: The First Course in the Finite Element, Logan, 4th Edition

5. The cantilevered beam depicted in the figure is subjected to two-plane bending. The loads are applied such that the planes of bending correspond to the principal moments of inertia. Noting that no axial or torsional loadings are present, model the beam as a single element (that is, construct the 8 × 8 stiffness matrix containing bending terms only) and compute the deflections of the free end, node 2. Determine the exact location and magnitude of the maximum bending stress. (Use E = 207 GPa.)

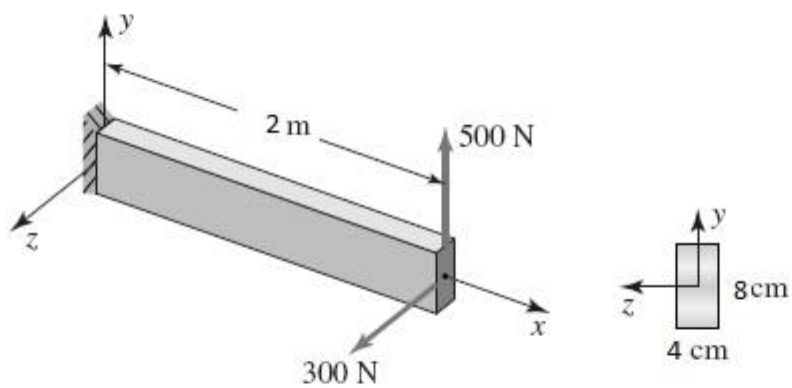


Figure P 5

Ref: Fundamentals of Finite Element Analysis, David V. Hutton, 2004